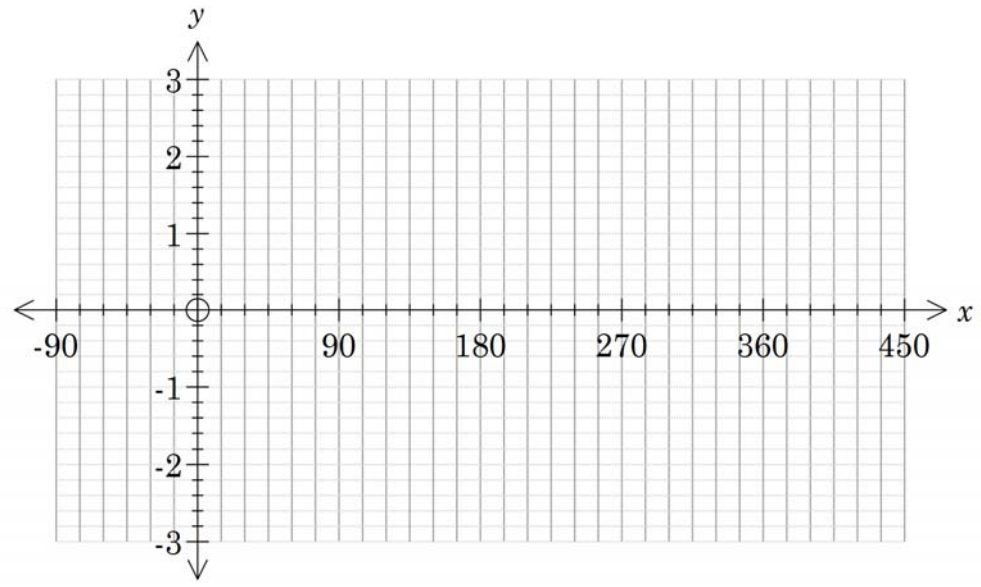
Activity 23 Auxiliary angle

Aim: Investigate the auxiliary angle technique to convert trigonometric sums to a different form.

1. Consider the function $y = \sqrt{3}\cos(x) + \sin(x)$.

   a) Draw a graph of the function in the Graph&Table application and sketch the graph on the axes below.



   b) Determine the period of the function.

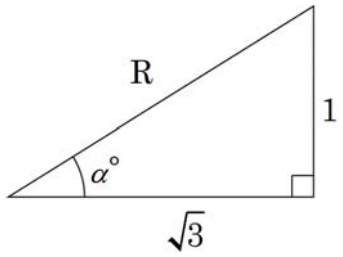
   c) Determine the maximum value of the function and the first positive x value for which this maximum occurs.

   d) Hence write the function in the form $y = R\cos(x - \alpha)$.

Your answers to Q1 should suggest to you that expressions of the form $a\cos(x)+b\sin(x)$ can be written in the form $R\cos(x-\alpha)$. But why should this be the case?

The answer lies in the compound angle formula.

2. Consider the triangle below:



a) Determine:

(i) R

(ii) $\cos(\alpha)$

(iii) $\sin(\alpha)$

(iv) α

b) In Main, enter the command tExpand$(R \times \cos(x - \alpha))$ and write the output below.

c) Use your answers to a) and b) to complete the following:

$\sqrt{3}\cos(x) + \sin(x)$

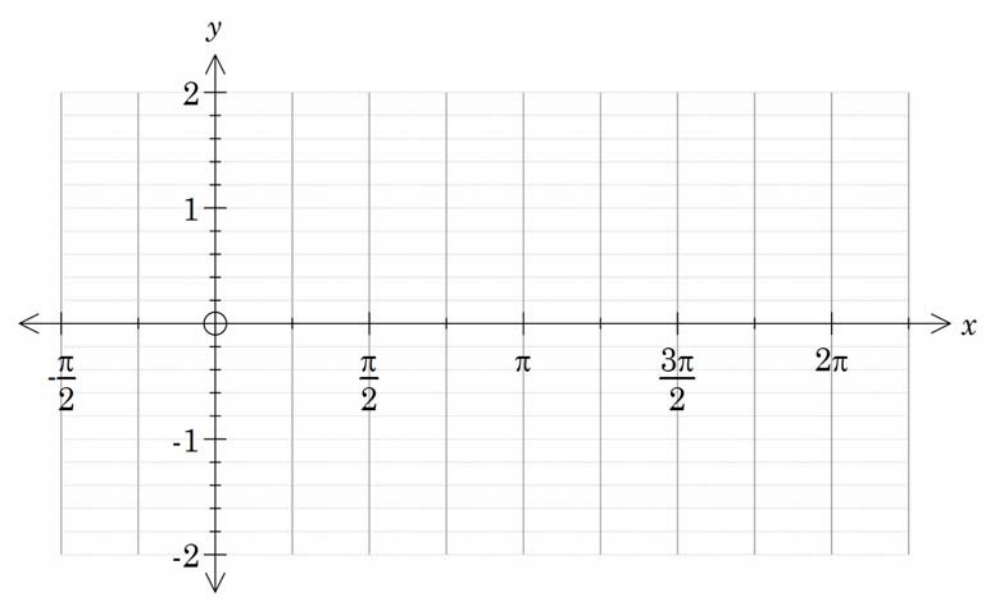
$= 2\left(\frac{\sqrt{3}}{2}\cos(x) + \frac{1}{2}\sin(x)\right)$

$=$

3. Consider the equation $y = \sin(x) - \cos(x)$.

a) Show that $\sin(x) - \cos(x) = \sqrt{2}\sin\left(x - \frac{\pi}{4}\right)$.

b) Hence sketch the graph of $y = \sin(x) - \cos(x)$ on the axes below.



c) Solve the equation $\sin(x) - \cos(x) = -1 \quad 0 \le x \le 2\pi$:

(i) Graphically

(ii) Algebraically

(iii) Using CAS in Main.

Learning notes

In general, sums and differences of the form $a\cos(x) \pm b\sin(x)$ can be converted to $R\cos(x \mp \alpha)$ where $R = \sqrt{a^2 + b^2}$ and $\alpha = \tan^{-1}\left(\frac{b}{a}\right)$, or equally $R\sin(x \pm \alpha)$ where $R = \sqrt{a^2 + b^2}$ and $\alpha = \tan^{-1}\left(\frac{a}{b}\right)$.

Prior to graphing calculators, the auxiliary angle technique would have been useful for solving equations of the form $a\cos(x) \pm b\sin(x) = k$ or for graphing sums of sine and cosine terms, but it remains now as an interesting transformation of expressions using the compound angle identity.